Theory of ground ice stability in sublimation environments

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Permanently stable ground ice is found beneath a permanently frost free surface on Mars, and similar conditions exist in the Antarctic Dry Valleys. This phenomenon is due to a balance of the vapor pressure of the ice with the atmospheric humidity in the presence of large amplitude temperature oscillations. An exactly solvable model example shows that the fraction of time the atmosphere needs to be saturated to stabilize the ice decreases with temperature amplitude. It is estimated that for conditions that prevail on Mars today, the mean temperature needs to be about 5 K lower than the frost point temperature for ground ice to be stable. A decomposition method to evaluate the contribution of short term weather events to ground ice stability is developed; when applied to a study site in the Dry Valleys, it reveals that the coldest periods contribute most to stabilization.

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I. INTRODUCTION

A. Motivation

Two recent discoveries in earth and planetary science relate to the stability of ground ice with respect to sublimation. One fifth of the surface of Mars is underlain by ground ice, which is thought to be permanently stable due to a balance between the atmospheric humidity and the ice, although the overlying planetary surface is perennially frost free [1,2]. Similarly, in the Dry Valleys of Antarctica, ground ice is found with an exceptionally old ("fossil") age, although there is no perennial ice on the surface and little seasonal precipitation [3,4]. The present paper tries to illuminate the reasons for this phenomenon, long-lived ice beneath dry surfaces. Despite the problem's relevance for understanding ice ages on Mars and its usefulness for inferences about the history of the East Antarctic ice sheet, there is to this date no mathematical study of this phenomenon. Numerical investigations have been carried out both for Mars, e.g., [5–8], and the Dry Valleys [9–11].

B. Basic equations and averaging procedure

Throughout this paper we consider a one-dimensional model, where a layer of ice is covered by a layer of soil that contains no perennial ice. Temperature oscillations decay with depth, and if the ice is very deeply buried its temperature is constant. (The annual thermal skin depth of dry soils is typically on the order of 1 m.) Vapor can migrate from the ice to the atmosphere and in the other direction, see Fig. 1 for illustration. The surface experiences the largest temperature variations, and the air may transition between saturated and unsaturated. Atmospheres can keep surplus water aloft for short time periods (e.g., overnight), but necessarily precipitate snow or frost onto the surface when saturated for a prolonged period of time.

H_2O transport in and out of the soil is in the form of vapor diffusion. The diffusive mass flux is [12]

$$J = - D\rho_0 \frac{\partial}{\partial z}\left(\frac{\rho_v}{\rho_0}\right), \tag{1}$$

where ρ_0 is the total air density, ρ_v is the mass density of water vapor, and D is the diffusion coefficient of the porous soil. In addition there is advective transport which is known to be of order $O(\rho_v/\rho_0)$ [13]. A reasonable approximation is $J=-D\partial\rho_v/\partial z$. For constant diffusivity, the time average is

$$\langle J\rangle = - D\frac{\partial\langle\rho_v\rangle}{\partial z}. \tag{2}$$

The physical phases involved are water vapor, ice, and adsorbed H_2O. Adsorption on soil surfaces occurs because the binding energy and entropy on a grain surface differs from that on a pure ice surface. At 200 K the vapor density is 10^9 times smaller than that of solid ice; although adsorbed H_2O is less dense than the solid form, it is still a huge source or sink compared to the vapor density. Dynamically, adsorption leads to a diffusion skin effect analyzed in Refs. [14,15].

It is possible to extract relevant quantities using an averaging procedure, without solving the diffusion equation and without detailed knowledge of adsorption capacity [8,9]. The mass conservation law is

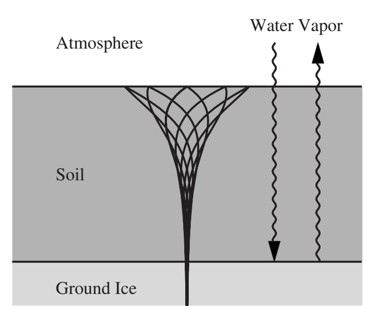

FIG. 1. Schematic model system where ground ice exchanges water vapor with the atmosphere through a layer or porous soil. Temperature oscillations decay with depth, as illustrated by a set of instantaneous temperature profiles.

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$$\frac{\partial}{\partial t}(\rho_v + \rho_{\rm ice} + \rho_{\rm adsorbed}) = -\frac{\partial J}{\partial z}. \qquad (3)$$

ρ_v is periodic in time, so its long term change is zero. Likewise, $\rho_{\rm adsorbed}$ is determined by temperature and partial pressure. Both quantities return to about the same values after a year. (Even if adsorbate mass does not reach equilibrium but is kinetically limited [16], periodic cycles yield little net effect.) And, at depths where ice is not permanent, $\partial\langle\rho_{\rm ice}\rangle/\partial t = 0$, for the same reason. We are left with $\partial\langle J\rangle/\partial z = 0$. The averages are over at least one solar year, or an integer multiple of periodic temperature cycles. The mean vapor flux $\langle J\rangle$ is constant with depth and determined by the boundary values

$$\langle J\rangle\frac{\Delta z}{D} = \langle\rho_v(\text{surface})\rangle - \langle\rho_v(\text{ice table})\rangle, \qquad (4)$$

where Δz is the thickness of the dry layer. Hence the loss or gain can be determined from averaging vapor densities at the boundaries (the surface and the ice table), irrespective of adsorbed water and transient subsurface frost condensation. Ground ice is stable (survives indefinitely) when $\langle J\rangle \geqslant 0$, unstable when $\langle J\rangle < 0$, and in equilibrium when $\langle J\rangle = 0$.

At any depth, there is either net depletion or net accumulation, such that the soil layer will remain free of permanent ice or pore spaces will fill with ice. For the environments considered here, significant changes in ice volume require at least many thousands of years. This slow evolution justifies to neglect the latent heat of sublimation compared to the heat available. For the same reason, the movement of the ice table or the change in pore ice fraction over one temperature cycle is tiny.

II. STABILITY CRITERION FOR EXACTLY SOLVABLE MODEL

A. Model setup

We consider a sinusoidal temperature oscillation with a mean T_m, amplitude T_a, and angular frequency ω,

$$T(t) = T_m + T_a\cos(\omega t). \qquad (5)$$

The saturation vapor pressure is given by

$$p_{\rm sv}(T) = p_m \exp\left[-\frac{H}{R}\left(\frac{1}{T} - \frac{1}{T_m}\right)\right], \qquad (6)$$

where $p_m = p_{\rm sv}(T_m)$, $H = 51.058$ MJ/kg is the sublimation enthalpy, and R is the universal gas constant. The partial pressure p in the atmosphere is assumed constant, unless saturated $p = \min(p_f, p_{\rm sv}(T))$. We can define a frost point temperature T_f, below which saturation occurs, $p_f = p_{\rm sv}(T_f)$. Since the temperature is highest at $t=0$, the atmosphere is unsaturated between a time $-t_1$ and t_1 and saturated during the remainder of the cycle.

It is convenient to introduce a parameter

$$\epsilon = \cos(\omega t_1), \qquad (7)$$

such that

$$\epsilon = \frac{T_f - T_m}{T_a}. \qquad (8)$$

The vapor density on the surface during unsaturated periods is

$$\rho_u = \frac{\mu}{R}\frac{p_f}{T}, \qquad (9a)$$

and when saturated

$$\rho_s = \frac{\mu}{R}\frac{p_{\rm sv}(T)}{T}, \qquad (9b)$$

where μ is the molar weight of H_2O, $\mu = 18$. We consider the limit when ice is very deeply buried, because otherwise the thermal conductivity of the ice would change the temperature profile and complicate the problem. The ground ice then is at the mean temperature. Without geothermal heating, ice is unstable unless it is stable at depth (although this has been disputed by numerical calculations in Ref. [17]). The vapor density at the ice is

$$\rho_g = \frac{\mu}{R}\frac{p_m}{T_m}. \qquad (9c)$$

The net flux over one cycle is

$$\langle J\rangle\frac{\Delta z}{D} = 2\int_0^{t_1}\rho_u dt + 2\int_{t_1}^{\pi/\omega}\rho_s dt - \int_0^{2\pi/\omega}\rho_g dt. \qquad (10)$$

The ice is stable when $\langle J\rangle \geqslant 0$ (inward flux) and unstable when $\langle J\rangle < 0$ (outward flux).

B. First order (analytically)

Expansion of Eqs. (6), (9a), and (9b) to first order in T_a around T_m and p_m yields

$$\rho_u = \frac{\mu}{R}\frac{p_m}{T_m^3}\left(T_m^2 + \frac{H}{R}T_a\cos(\omega t_1) - T_a T_m\cos(\omega t)\right), \qquad (11a)$$

$$\rho_s = \frac{\mu}{R}\frac{p_m}{T_m^3}\left[T_m^2 + T_a\left(\frac{H}{R} - T_m\right)\cos(\omega t)\right]. \qquad (11b)$$

The net flux over one cycle (10) is

$$\langle J\rangle = \frac{D}{\Delta z}\frac{\mu}{R}\frac{2p_m T_a}{T_m^3\omega}\frac{H}{R}[\omega t_1\cos(\omega t_1) - \sin(\omega t_1)]. \qquad (12)$$

The flux vanishes when $\omega t_1 = \tan(\omega t_1)$. The only solution with $|\omega t_1| \leqslant \pi$ is $t_1 = 0$. Consequently, the condition for equilibrium is $\epsilon = 1$ or

$$T_f = T_m + T_a. \qquad (13)$$

The lowest order balance is not $T_f = T_m$, as some authors assume. The condition for stability is $T_f \geqslant T_m + T_a$. (But, as will become clear below, this practically overestimates the requirement.) To first order in temperature amplitude, the atmosphere needs to be saturated all the time for ice to be stable. A continuously saturated atmosphere would have to precipitate, meaning that the first order balance does not explain permanent ice beneath a permanently dry surface.

C. Second order (analytically)

Although T_a is small in the perturbation expansion, $T_f - T_m$ determines t_1, which can vary from 0 to π. Hence the strategy is to first expand in T_a, keeping t_1 as a variable, and later expand t_1 around the first order solution.

Expansion of Eqs. (9a) and (9b) to second order in T_a yields

$$\rho_u = \frac{\mu}{R}\frac{p_m}{2T_m^5}\Bigg[2T_m^4 + 2\frac{H}{R}T_aT_m^2\epsilon + \left(\frac{H}{R}\right)^2 T_a^2\epsilon^2 - 2\frac{H}{R}T_a^2T_m\epsilon^2 - 2T_aT_m\left(T_m^2 + \frac{H}{R}T_a\epsilon\right)\cos(\omega t) + 2T_a^2T_m^2\cos^2(\omega t)\Bigg], \tag{14a}$$

$$\rho_s = \frac{\mu}{R}\frac{p_m}{2T_m^5}\Bigg\{2T_m^4 + 2T_a\left(\frac{H}{R} - T_m\right)T_m^2\cos(\omega t) + T_a^2\Bigg[\left(\frac{H}{R}\right)^2 - 4\frac{H}{R}T_m + 2T_m^2\Bigg]\cos^2(\omega t)\Bigg\}. \tag{14b}$$

After integration and some algebra,

$$\langle J\rangle = \frac{D}{\Delta z}\frac{\mu}{R}\frac{p_mT_a}{4T_m^5\omega}\Bigg\{2\pi T_a\Bigg[\left(\frac{H}{R}\right)^2 - 4\frac{H}{R}T_m + 2T_m^2\Bigg] + 2\frac{H}{R}\omega t_1\left(\frac{H}{R}T_a(-1+2\epsilon^2) + 4T_m(T_a + T_m\epsilon - T_a\epsilon^2)\right) - 8\frac{H}{R}T_m(T_m + T_a\epsilon)\sin(\omega t_1) - \frac{H}{R}T_a\left(\frac{H}{R} - 4T_m\right)\sin(2\omega t_1)\Bigg\}. \tag{15}$$

Since the flux is set to zero for equilibrium, a factor $\mu p_m T_a D/(2\omega RT_m^5\Delta z)$ can be canceled, and t_1 can be expressed in terms of ϵ with Eq. (7),

$$0 = \pi T_a\Bigg[\left(\frac{H}{R}\right)^2 - 4\frac{H}{R}T_m + 2T_m^2\Bigg] + \frac{H}{R}\left(\frac{H}{R}T_a(-1+2\epsilon^2) + 4T_m(T_a + T_m\epsilon - T_a\epsilon^2)\right)\arccos\epsilon - 4\frac{H}{R}T_m(T_m + T_a\epsilon)\sqrt{1-\epsilon^2} - \frac{H}{R}T_a\left(\frac{H}{R} - 4T_m\right)\epsilon\sqrt{1-\epsilon^2}. \tag{16}$$

This is an implicit equation for ϵ, but to obtain an explicit solution, further series expansion is necessary. The first-order calculation has shown that t_1 lies close to 0, not close to $\pi/2$, for oscillations of small amplitude. Expansion around $\epsilon=1$ yields

$$0 = \pi T_a\Bigg[\left(\frac{H}{R}\right)^2 - 4\frac{H}{R}T_m + 2T_m^2\Bigg] + \frac{8\sqrt{2}}{3}\frac{H}{R}\left(-\frac{H}{R}T_a + 3T_aT_m - T_m^2\right)(1-\epsilon)^{3/2} + O(1-\epsilon)^{5/2}. \tag{17}$$

Omitting the higher orders in $(1-\epsilon)$ leads to

$$(1-\epsilon)^{3/2} = \frac{3\pi}{8\sqrt{2}}T_a\frac{(H/R)^2 - 4(H/R)T_m + 2T_m^2}{(H/R)[(H/R)T_a - 3T_aT_m + T_m^2]}. \tag{18}$$

If we further take advantage of $T_a \ll T_m$, the equilibrium condition is

$$\frac{T_f - T_m}{T_a} = 1 - \left[\frac{3\pi}{8\sqrt{2}}\frac{T_a}{T_m}\left(\frac{H}{RT_m} - 4 + 2\frac{RT_m}{H}\right)\right]^{2/3} \tag{19}$$

and, using $\omega t_1 = \arccos\epsilon \approx \sqrt{2(1-\epsilon)}$,

$$\omega t_1 = \left[\frac{3\pi}{4}\frac{T_a}{T_m}\left(\frac{H}{RT_m} - 4 + 2\frac{RT_m}{H}\right)\right]^{1/3}. \tag{20}$$

Practically, however, $H/R \approx 30T_m$ and a better approximation of Eq. (18) may be

$$\frac{T_f - T_m}{T_a} = 1 - \left(\frac{3\pi}{8\sqrt{2}}T_a\frac{(H/R) - 4T_m}{(H/R)T_a + T_m^2}\right)^{2/3}, \tag{21}$$

$$\omega t_1 = \left[\frac{3\pi}{4}T_a\frac{(H/R) - 4T_m}{(H/R)T_a + T_m^2}\right]^{1/3}. \tag{22}$$

The fraction of time during each cycle the atmosphere is unsaturated is $\omega t_1/\pi$. Hence the duration it is unsaturated increases with temperature amplitude, if ground ice and atmosphere are to be exactly in equilibrium.

D. All orders (numerically)

Numerically, the flux (10) can be evaluated without linearization. Figure 2 shows the equilibrium solution, obtained by numerically finding the root $\langle J\rangle=0$ (solid line). The graph confirms that the perturbation in T_a is singular (not analytic), because it has infinite slope at $T_a=0$. The dashed and dotted lines show the approximations derived above. An amplitude of only a few K allows the atmosphere to be unsaturated for a sizable fraction of time while still balancing the vapor pressure of the ice. This is a consequence of the strong nonlinear dependence of vapor pressure on temperature. Although the absolute humidity required to stabilize ice increases with temperature amplitude, the fraction during which the air needs to be saturated decreases with amplitude.

Another result from numerical evaluation of the flux for a sinusoidal temperature oscillation is shown in Fig. 3. Assuming a frost point temperature of 200 K, close to conditions on present-day Mars, the mean temperature and temperature amplitude required for ground ice stability are drawn as a solid line. This dependence is very nonlinear in $T_f - T_m$, which allows us to estimate how much lower T_m has to be than T_f without accurate knowledge of T_a. Day-night temperature differences on Mars are often of order 80 K, and as a rule of thumb, the mean temperature should be about 5 K

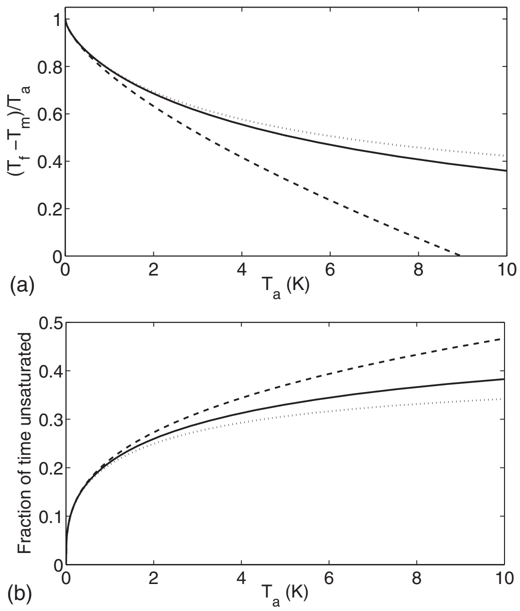

FIG. 2. (a) Required frost point temperature T_f as a function of temperature amplitude T_a. The mean temperature is $T_m=200$ K. The solid line shows the numerical solution, the dashed line is the perturbation expansion (19), and the dotted line is approximation (21). (b) The maximum fraction of time the atmosphere can be unsaturated and still stabilize the ground ice, $\pi t_1/\omega$.

below the frost point temperature. This is consistent with model calculations using detailed temperature histories for Mars, where a difference of at least 6 K has been reported [8].

III. IDENTIFICATION OF STABILIZING EVENTS

Realistic environments involve variable humidity, and it would be useful to identify which weather events or seasons are most favorable for the survival of ground ice. For example, Föhns increase air temperature and change air moisture [18]. According to Eq. (4), ice loss increases when $\langle\rho_v(\text{surface})\rangle-\langle\rho_v(\text{ice table})\rangle$ is smaller. A warm humid event would increase $\langle\rho_v(\text{surface})\rangle$, but also increase subsurface temperatures and therefore $\langle\rho_v(\text{ice table})\rangle$, and the sign of its effect is unclear.

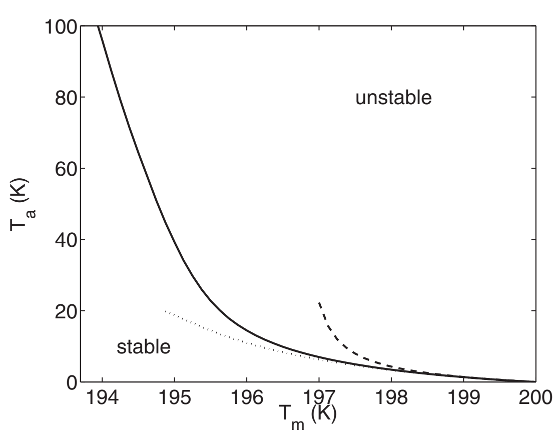

FIG. 3. Required mean temperature T_m and temperature amplitude T_a for a frost point temperature of $T_f=200$ K. The required mean temperature is only a few K less than T_f, even for large amplitudes. The dashed and dotted lines show approximations (19) and (21), respectively.

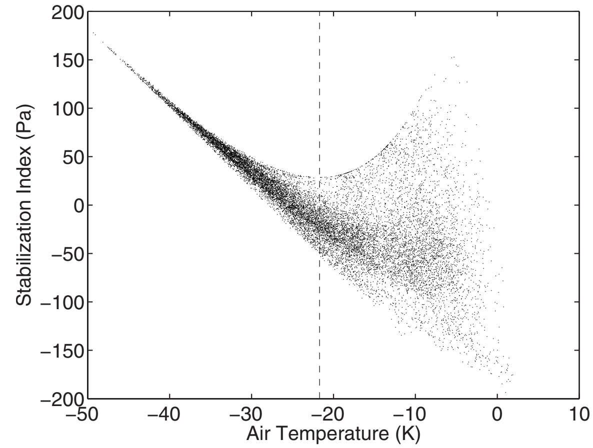

FIG. 4. Stabilization index, which is a measure of the contribution of an instantaneous event to the humidity balance between the atmosphere and the ice, for meteorological data from Beacon Valley, Antarctica [19] from 2001 to 2004. A positive index indicates an above average contribution to stabilization. The vertical dashed line shows the mean temperature.

In the following analysis, the net vapor flux is taken to be proportional to gradients in partial pressure p instead of the vapor density ρ_v, which is approximately equivalent, but more clearly untangles pressure and temperature dependence.

Consider an event that lasts a short time interval dt. The increase in ice (mean) temperature due to the heat input is $dT=(T-T_m)dt$, and the resulting increase of the vapor pressure of the ice is $dp_g=p_{sv}(T_m+dT)-p_{sv}(T_m)=(\partial p_{sv}/\partial T)dT$, where T_m is the mean temperature. (In a stationary situation, the mean temperature of the soil and of the ice are the same, because the mean heat flux vanishes.) On the other hand, the extra contribution to the humidity in the air is $dp_a=(p-p_m)dt$. One can define a stability index,

$$i=\frac{dp_a-dp_g}{dt}=p-p_m-\left.\frac{\partial p_{sv}}{\partial T}\right|_{T_m}(T-T_m). \qquad (23)$$

Here, p and T are functions of time. The stability index averages to zero, $\langle i\rangle=0$, and indicates whether any interval of time contributes more or less than the average to stabilization. (But the average condition may or may not be stable; this index is only useful close to stability.)

Figure 4 shows this stability index for data from a meteorological station on the floor of Beacon Valley, Antarctica [19], where ground ice is unstable but close to stable [10,11]. This environment is 29 Pa short of stability, $p_{sv}(T_m)-p_m=29$ Pa. Measurements are available every 15 min, but only every tenth data point is shown to avoid overcrowding the plot.

There is an upper bound to the stabilization index, which corresponds to a relative humidity of 100%, and a lower bound at a humidity of 0%. The upper bound reaches a minimum for $T=T_m$, where $i=p_{sv}(T_m)-p_m$. Warm humid periods are strongly stabilizing, while warm dry periods are the most destabilizing of all. The individual events with the highest stabilization index at above average temperature occurred on January 24, 2003 at an air temperature around −5 °C, relative humidities around 90%, and wind velocities considerably lower than average.

Also favorable to the persistence of ice are the coldest events. Although they provide a negligible amount of moisture, they draw heat from the ground and reduce the mean temperature and thus the vapor pressure of the ground ice.

The plot also reveals again that large amplitude oscillations are crucial. The largest contributions to stability occur at temperature extremes. At mean temperature, i can be at most 29 Pa, no more than required for stability. Without large excursions from the mean, the relative humidity would have to be close to 100% all the time to balance the vapor pressure of the ice. For large temperature amplitude, on the other hand, the relative humidity never needs to reach 100% at any time.

The stability index can be generalized to a situation where the ground ice is not very deeply buried but experiences temperature fluctuations. Instead of evaluating the derivative of the vapor pressure at T_m in Eq. (23), it should be evaluated at a temperature T^* defined by $\langle p_{sv}(T)\rangle = p_{sv}(T^*)$, where the average is formed at the depth of the ice table.

IV. CONCLUSIONS

For a simple model with sinusoidal temperature oscillations, deeply buried ice, and constant absolute humidity, we derived an asymptotic formula for the frost point temperature of the atmosphere required to balance the vapor pressure of the ice, Eq. (21). It shows a rapid nonanalytic dependence on temperature amplitude. The fraction of time the atmosphere needs to be saturated to balance the vapor pressure of the ice decreases with temperature amplitude.

Numerical solutions of the same idealized model indicate that for conditions typical of Mars, the mean temperature must be about 5 K less than the frost point temperature to stabilize ground ice.

A method is described to determine the contribution of weather events to ground ice stability. In Beacon Valley, the coldest periods contribute most to stabilization; the reduction in ice temperature outweighs the deficit in humidity. Hence one can speculate that additional exceptionally cold events would efficiently reduce or even prevent ice loss.

An insight from the perturbation expansion as well as the analysis of stabilization indices is that large temperature oscillations are crucial for the persistence of ice beneath a dry surface. Far-from-average air temperatures are capable of balancing the vapor pressure of the ice with only brief condensation or even without condensation in the atmosphere.

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